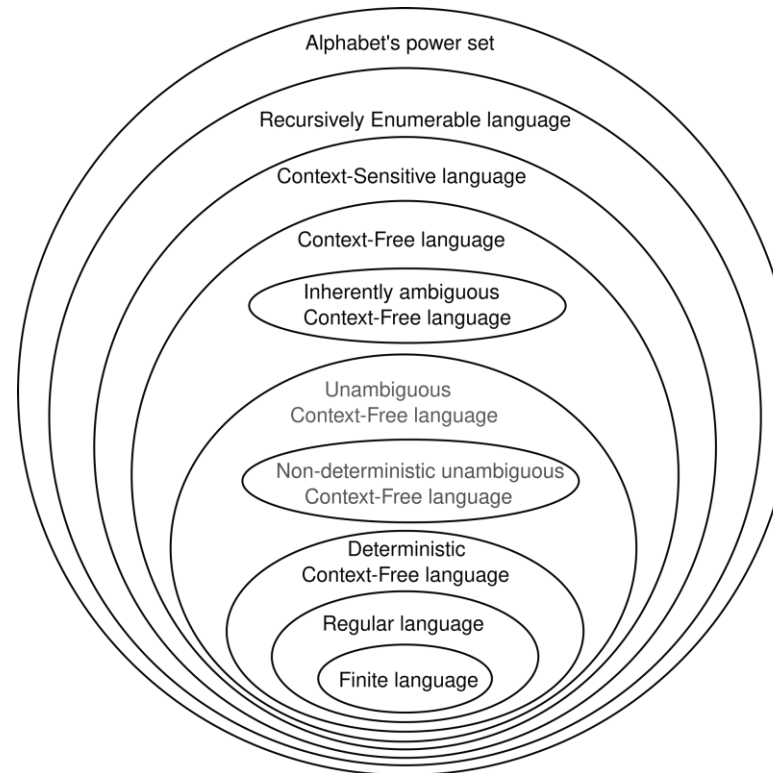


Formal languages

Tiny introduction



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Lecture outline

- regular expressions (here just for your quick revision)
- context dependent grammars
- Chomsky hierarchy

Regular expressions - a quick resume 1/3

- standard notation for characterizing text sequences
- used in all kinds of text processing and information extraction tasks
- many different syntaxes (Perl, grep, sed, awk, Python, etc)
- let's use regular expressions (RE) from python
- if A and B are REs then AB is RE
- a,b,...,z, A, B,... Z,0,1,...,9 are REs
- e.g. abceda is RE
- . matches any character, e.g.: va.a matches vaba or vaza or vaya
- ^ matches the start of a string; ^.oga matches noga or joga, but not nadloga
- \$ matches the end of a string
- * matches 0 or more repetitions of the previous RE: ab* matches a, ab, abb, ...
- + matches 1 or more repetitions of the previous RE: ab+ matches ab, abb, ... but not a

Regular expressions 2/3

- ? matches 0 or 1 repetitions of the previous RE: `ab?` matches `a` or `ab`
- *, + and ? are greedy: they match the longest possible string, e.g., `<.*>` on the string `<a> b <c>` matches the whole string
- *?, +?, ?? cause minimal matching of *, +, and ?, e.g., `<.*?>` on the string `<a> b <c>` will match `<a>`
- {m} matches m repetitions of a previous RE: `b{5}` matches only `bbbbbb`
- {m,n} matches from m to n repetitions of a previous RE
- {,n} is the same as {0,n}
- {m,} is the same as {m,∞}
- {m,n}? is a non-greedy variant of {m,n}
- \ is an escape character, it makes the next character special, e.g.,
 - `\\` matches `\`
 - `\*` matches `*`

Regular expressions 3/3

- `[]` represents a set of characters, e.g., `[abc]` matches a, b, or c;
with `[]` we can represent a sequence of characters, e.g., `[a-z]` matches all lowercase letters from a to z
special characters inside the set are not special, e.g., `?,+,*`
- `[^]` (^ as the first character) represents a complement of a set, e.g., `[^abc]` matches all characters except a, b, and c
- `|` in `A|B`, where A and B are REs, means that RE matches A or B, several REs separated with `|` is tested from left to right,
operator `|` is not greedy
- `(...)` matches RE in the parenthesis and marks a group, which can be used later or retrieved with `\group_number`
- `(?aiLmsux)`, where after `?` there are one or more letters means:
a – only ASCII matches, i – ignore lower/uppercase, L – depend on the local settings,
m – multi-line, s – the dot matches everything, etc. – check the manual
- many other useful details

Example

- Find me all instances of the word “the” in a text.

the

Misses capitalized examples

[tT]he

Incorrectly returns other or theology

[^a-zA-Z][tT]he[^a-zA-Z]

Errors

- The process we just went through was based on **fixing two kinds of errors:**
 1. Matching strings that we should not have matched (**there, then, other**)
False positives (Type I errors)
 2. Not matching things that we should have matched (The)
False negatives (Type II errors)

Errors cont.

- In NLP we are always dealing with these kinds of errors.
- Reducing the error rate for an application often involves two antagonistic efforts:
 - Increasing accuracy or precision (minimizing false positives)
 - Increasing coverage or recall (minimizing false negatives).

Substitutions

- Substitution in Python and UNIX commands:
- `s/regex1/pattern/`
- e.g.:
- `s/colour/color/`

Simple Application: ELIZA

```
=====
EEEEEEEE L      IIIIII ZZZZZZZ      AAA
E        L      I      Z      A      A
E        L      I      Z      A      A
EEEEEE   L      I      Z      A      A
E        L      I      Z      A      A
E        L      I      Z      A      A
EEEEEEEE LLLLLLLL IIIIII ZZZZZZ      A      A
=====
ELIZA > (PLEASE TYPE IN ALL CAPS) WHAT'S YOUR NAME DEAR ?
AMIT
ELIZA > HI, I'M ELIZA. WHAT DO YOU WANT TO TALK ABOUT ?
AMIT > CAN I TALK ABOUT MY PROBLEM ?
ELIZA > SURE... ?
AMIT > DUE TO ON GOING EXAMS IN COLLEGE, I HAVE A LOT OF STRESS.
ELIZA > PLEASE RELAX A LITTLE. SLEEP WELL.
AMIT > THANKS FOR YOUR ADVICE.
ELIZA > NO MENTION.
AMIT > BYE
ELIZA > BYE AND KEEP IN TOUCH...
=====
```

- Early NLP system that imitated a Rogerian psychotherapist (Weizenbaum, 1966).
- Uses pattern matching to match, e.g.,:
 - "I need X"and translates them into, e.g.
 - "What would it mean to you if you got X?"

Simple Application: ELIZA

Men are all alike.

IN WHAT WAY

They're always bugging us about something or other. CAN YOU
THINK OF A SPECIFIC EXAMPLE

Well, my boyfriend made me come here.

YOUR BOYFRIEND MADE YOU COME HERE

He says I'm depressed much of the time.

I AM SORRY TO HEAR YOU ARE DEPRESSED

How ELIZA works

- s/. * I'M (depressed|sad) . */I AM SORRY TO HEAR YOU ARE \1/
- s/. * I AM (depressed|sad) . */WHY DO YOU THINK YOU ARE \1/
- s/. * all . */IN WHAT WAY?/
- s/. * always . */CAN YOU THINK OF A SPECIFIC EXAMPLE?/

Summary

- Regular expressions play a surprisingly large role
 - Sophisticated sequences of regular expressions are often the first model for any text processing text
- For hard tasks, we use machine learning classifiers
 - But regular expressions are still used for pre-processing, or as features in the classifiers
 - Can be very useful in capturing generalizations

RE exercises

Write regular expressions for the following languages

- the set of all alphabetic strings;
- the set of all lower case alphabetic strings ending in a b
- the set of all strings with two consecutive repeated words (e.g., “Humbert Humbert” and “the the” but not “the bug” or “the big bug”);
- the set of all strings from the alphabet a,b such that each a is immediately preceded by and immediately followed by a b;
- all strings that start at the beginning of the line with an integer and that end at the end of the line with a word;
- all strings that have both the word grotto and the word raven in them (but not, e.g., words like grottos that merely contain the word grotto);

Formal Languages and Models

- **Language:** a (possibly infinite) set of strings made up of symbols from a finite alphabet
- **Model** of a language: can *recognize* and *generate* **all** and **only** the strings from the language
 - Serves as a definition of the formal language
- Alphabet Σ is a finite set of symbols, e.g., $\Sigma = \{0,1\}$ or $\Sigma = \{a,b,c,d\}$.
- String is a sequence of symbols from alphabet
- ϵ is an empty set
- $\Sigma \cup \Sigma\Sigma$ is a set of all strings of length 1 or 2
- Σ^* is a set of all strings from alphabet
- imprecise notation, e.g., 0 is a symbol and 0 is a string, depending on the context

Merrill, W., 2021. Formal Language Theory Meets Modern NLP. [arXiv preprint arXiv:2102.10094](https://arxiv.org/abs/2102.10094).

About formal languages and their relation with neural networks.

Language

- Language is a subset of Σ^* for an alphabet Σ .
- Example: language of 0 and 1, where there are no two consecutive 1s
- $L = \{\epsilon, 0, 1, 00, 01, 10, 000, 001, 010, 100, 101, 0000, 0001, 0010, 0100, 0101, 1000, 1001, 1010, \dots\}$

Chomsky Hierarchy

- Regular language
 - Model: regular expressions, finite state automata
- Context free language
- Context sensitive language
- Unrestricted language
 - Model: Turing Machine

Regular Expressions and Languages

- A regular expression pattern can be mapped to a set of strings
- A regular expression pattern defines a language (in the formal sense)
 - the class of this type of languages is called a **regular language**

An example of non-regular language

$$L_1 = \{0^n 1^n \mid n \geq 1\}$$

$$L_1 = \{01, 0011, 000111, \dots\}$$

An example

$L_2 = \{w \mid w \in \{(\, , \,)\}^* \text{ with balanced brackets}\}.$

E.g.: $()$, $()()$, $(())$, $(())()$,...

Context Free Grammars (CFG)

- A *context-free grammar* is a notation for describing languages.
- It is more powerful than finite automata or RE's, but still cannot define all possible languages.
- Useful for nested structures, e.g., parentheses in programming languages.
- Basic idea is to use “variables” to stand for sets of strings (i.e., languages).
- These variables are defined recursively, in terms of one another.
- Recursive rules (“productions”) involve only concatenation.
- Alternative rules for a variable allow union.

Example: CFG for $\{ 0^n 1^n \mid n \geq 1 \}$

- Productions:

$S \rightarrow 01$

$S \rightarrow 0S1$

- 01 is part of a language
- if w is in the language, so is $0w1$

Syntax

- Syntax = rules describing how words can connect to each other
- *that and after year last*
- *I saw you yesterday*
- *colorless green ideas sleep furiously*
- the kind of implicit knowledge of your native language that you had mastered by the time you were 3 or 4 years old without explicit instruction
- not necessarily the type of rules you were later taught in school.

Syntax

- Why should you care?
 - Grammar checkers
 - Question answering
 - Information extraction
 - Machine translation

CFG Formalism

- *Terminals* = symbols of the alphabet of the language being defined.
- *Variables* = *nonterminals* = a finite set of other symbols, each of which represents a language.
- *Start symbol* = the variable whose language is the one being defined.
- A *production* has the form *variable* $\rightarrow$ *string of variables and terminals*.
- *Convention*:
 - A, B, C,... are variables.
 - a, b, c,... are terminals.
 - ..., X, Y, Z are either terminals or variables.
 - ..., w, x, y, z are strings of terminals only.
 - α , β , γ ,... are strings of terminals and/or variables.

Example: Formal CFG

- Here is a formal CFG for $\{ 0^n 1^n \mid n \geq 1 \}$.
- Terminals = $\{0, 1\}$.
- Variables = $\{S\}$.
- Start symbol = S .
- Productions =
 - $S \rightarrow 01$
 - $S \rightarrow 0S1$

Derivations – Intuition

- We *derive* strings in the language of a CFG by starting with the start symbol, and repeatedly replacing some variable A by the right side of one of its productions.
 - That is, the “productions for A ” are those that have A on the left side of the $\rightarrow$.

Derivations – Formalism

- We say $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if $A \rightarrow \gamma$ is a production.
- **Example:** $S \rightarrow 01$; $S \rightarrow 0S1$.
- $S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000111$.

Iterated Derivation

- $\Rightarrow^*$ means “zero or more derivation steps.”
- **Basis**: $\alpha \Rightarrow^* \alpha$ for any string α .
- **Induction**: if $\alpha \Rightarrow^* \beta$ and $\beta \Rightarrow \gamma$, then $\alpha \Rightarrow^* \gamma$.

Example: Iterated Derivation

- Productions: $S \rightarrow 01$; $S \rightarrow 0S1$.
- Derivation: $S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000111$.
- So: $S \Rightarrow^* S$; $S \Rightarrow^* 0S1$; $S \Rightarrow^* 00S11$; $S \Rightarrow^* 000111$.

Language of a Grammar

- If G is a CFG, then $L(G)$, the *language of G* , is $\{w \mid S \Rightarrow^* w\}$.
 - **Note**: w must be a terminal string, S is the start symbol.
- **Example**: G has productions $S \rightarrow \epsilon$ and $S \rightarrow 0S1$.
- $L(G) = \{0^n 1^n \mid n \geq 0\}$.
- **Note**: ϵ is a legitimate right side.

Context-Free Languages

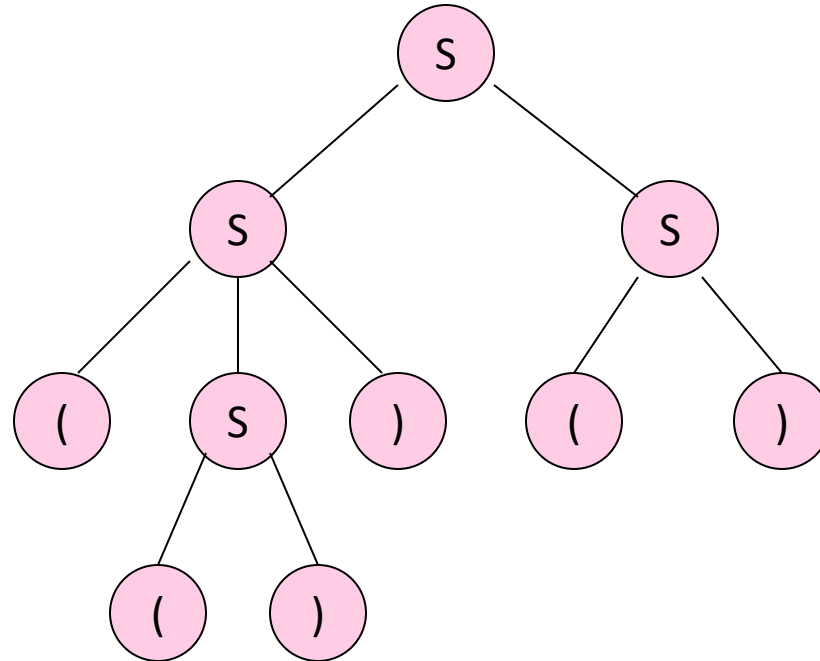
- A language that is defined by some CFG is called a *context-free language*.
- There are CFL's that are not regular languages, such as the example just given.
- But not all languages are CFL's.
- *Intuitively*: CFL's can count two things, not three.

Parse Trees

- *Parse trees* are trees labeled by symbols of a particular CFG.
- **Leaves**: labeled by a terminal or ϵ .
- **Interior nodes**: labeled by a variable.
 - Children are labeled by the right side of a production for the parent.
- **Root**: must be labeled by the start symbol.

Example: Parse Tree

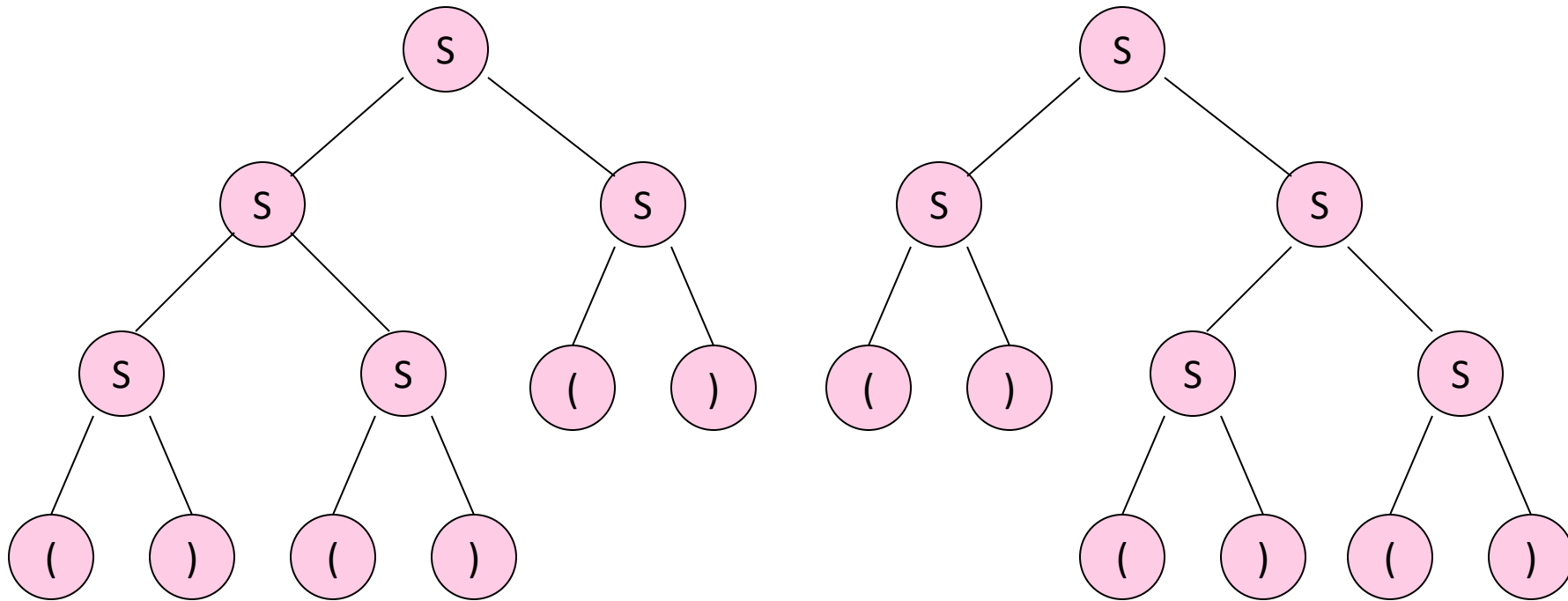
$S \rightarrow SS \mid (S) \mid ()$



Ambiguous Grammars

- A CFG is *ambiguous* if there is a string in the language that is the yield of two or more parse trees.
- Example: $S \rightarrow SS \mid (S) \mid ()$
- Two parse trees for $()()()$ on next slide.

Example



Ambiguity is a Property of Grammars, not Languages

- For the balanced-parentheses language, here is another CFG, which is unambiguous.

$B \rightarrow (RB \mid \epsilon$

B, the start symbol,
derives balanced strings.

$R \rightarrow) \mid (RR$

R generates strings that
have one more right bracket
than left.

Inherent Ambiguity

- It would be nice if for every ambiguous grammar, there were some way to “fix” the ambiguity, as we did for the balanced-parentheses grammar.
- Unfortunately, certain CFL’s are *inherently ambiguous*, meaning that every grammar for the language is ambiguous.

Example: Inherent Ambiguity

- The language $\{0^i1^j2^k \mid i = j \text{ or } j = k\}$ is inherently ambiguous.
- **Intuitively**, at least some of the strings of the form $0^n1^n2^n$ must be generated by two different parse trees, one based on checking the 0's and 1's, the other based on checking the 1's and 2's.

One Possible Ambiguous Grammar

$S \rightarrow AB \mid CD$

$A \rightarrow 0A1 \mid 01$

$B \rightarrow 2B \mid 2$

$C \rightarrow 0C \mid 0$

$D \rightarrow 1D2 \mid 12$

A generates equal 0's and 1's

B generates any number of 2's

C generates any number of 0's

D generates equal 1's and 2's

And there are two derivations of every string with equal numbers of 0's, 1's, and 2's. E.g.:

$S \Rightarrow AB \Rightarrow 01B \Rightarrow 012$

$S \Rightarrow CD \Rightarrow 0D \Rightarrow 012$

Exercises

- Write CFG for a language
- $L(G) = \{\text{all words of a form } a^n b^m c^k, \text{ where } n + m = k\}$
- $L(G) = \{\text{all words of a form } a^n b^m c^k, \text{ where } n + k = m\}$

Chomsky Normal Form

- A CFG is said to be in *Chomsky Normal Form* if every production is of one of these two forms:
 1. $A \rightarrow BC$ (right side is two variables).
 2. $A \rightarrow a$ (right side is a single terminal).
- **Theorem:** If L is a CFL, then $L - \{\epsilon\}$ has a CFG in CNF.

Decision properties of CFG

1. $w \in L$
2. $L = \{\}$
3. L is infinite
4. $L_1 = L_2$
5. $L_1 \cap L_2 = \{\}$

Algorithm CYK – testing membership

- CYK: Cocke – Younger – Kasami
- $\text{CFG} = \{V, T, S, P\}$
- answers the question $x \in L$ (or equivalently $S \Rightarrow^* x$)
- examples
 - is a given program correct according to the given grammar
 - is the given sentence grammatically correct
- requires CFG in Chomsky normal form
- $O(n^3)$, where $n = |w|$.

CYK Algorithm

- Let $w = a_1 \dots a_n$.
- We construct an n -by- n triangular array of sets of variables.
- $X_{ij} = \{\text{variables } A \mid A \Rightarrow^* a_i \dots a_j\}$.
- Induction on $j-i+1$.
 - The length of the derived string.
- Finally, ask if S is in X_{1n} .

CYK Algorithm – (2)

- **Basis:** $X_{ii} = \{A \mid A \rightarrow a_i \text{ is a production}\}$.
- **Induction:** $X_{ij} = \{A \mid \text{there is a production } A \rightarrow BC \text{ and an integer } k, \text{ with } i \leq k < j, \text{ such that } B \text{ is in } X_{ik} \text{ and } C \text{ is in } X_{k+1,j}\}$.

CYK example

- $S \rightarrow A B$
 $A \rightarrow BC \mid a$
 $B \rightarrow CC \mid b$
 $C \rightarrow a$
- ? $S \rightarrow aaab$

	a	a	a	b
1	A,C	A,C	A,C	B
2	B	B	S	
3	S,A	/		
4	S			

CYK exercises

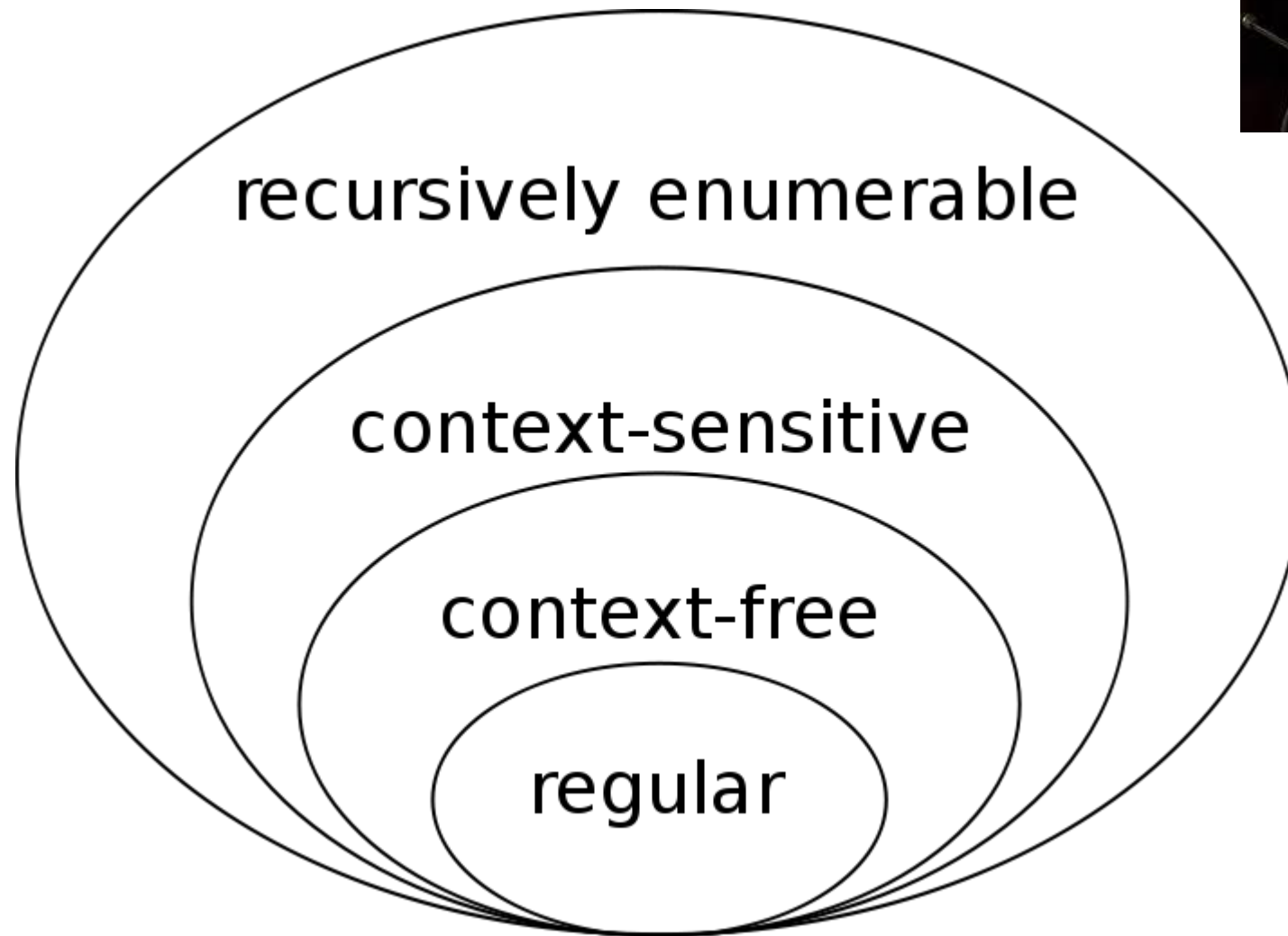
- $S \rightarrow P N \mid \text{other}$
 $P \rightarrow I E$
 $I \rightarrow \text{if}$
 $E \rightarrow \text{expression}$
 $N \rightarrow T S$
 $T \rightarrow \text{then}$
- is the sentence correct
 $S \rightarrow \text{if expression then if expression then other}$

- $S \rightarrow A C \mid B D \mid A E$
 $C \rightarrow B B$
 $D \rightarrow A A$
 $E \rightarrow B A \mid A B$
 $A \rightarrow a \mid A E \mid E A \mid B D$
 $B \rightarrow b \mid B E \mid E B \mid A C$
- ? $S \rightarrow \text{baabba}$

Tools for grammars

- gnu programs bison and yacc
- based on CFG, they generate a recognizer code in C, C++, or java

Chomsky hierarchy



Order 3

- Order 3 grammars are regular languages
- Grammars of the form

$S \rightarrow aA$

$S \rightarrow a$

Order 2

- CFGs
- Form $A \rightarrow \alpha$
- α is a string of terminals and nonterminals
- programming languages

Order 1

- Context dependent grammars CDG
- Form $\alpha A \beta \rightarrow \alpha \gamma \beta$
- A is a variable, α , β , and γ are strings of terminals and nonterminals
- α and β can be empty, γ has to be non-empty
- natural languages

Order 0

- Unbounded (Turing) grammars and Turing languages, i.e., languages recognizable by Turing machines
- Form $\alpha \rightarrow \beta$
- There are languages unrecognizable with Turing machines – diagonal proof